ON ZHUKOVSKY FUNCTION

Zhukovsky function

$$w = \frac{1}{2}\left(z + \frac{1}{z}\right) = \lambda(z)$$

maps conformally both the exterior and interior of the unit circle S^1 in the z-plane onto the complement to the interval $[-1, 1]$. It is a $2-1$ mapping of $S^1 \setminus \{1, -1\}$ onto $(-1, 1)$.

Since

$$\lambda'(z) = \frac{1}{2}\left(1 - \frac{1}{z^2}\right),$$

Zhukovsky function is no longer locally conformal at $z = \pm 1$. It follows from the formula

$$\frac{w-1}{w+1} = \left(\frac{z-1}{z+1}\right)^2,$$

that Zhukovsky map doubles the angles between two curves passing through $z = \pm 1$.

The inverse map, considered in Ahlfors, is given by a formula

$$z = w \pm \sqrt{w^2 - 1},$$

and maps the complement the interval $[-1, 1]$ onto the exterior of S^1 (sign 'plus') or onto the interior of S^1 (sign 'minus'). Here $\sqrt{w^2 - 1}$ is understood as the principal branch with the cut along $[-1, 1]$.

Let γ be a circle passing through -1 and 1 and having an angle $\alpha < \pi/2$ with the real axis. Its $2-1$ image is a curve connecting 1 and -1 in the upper half-plane, which has an angle 2α with the real axis at $z = 1$ (see Fig. 1). Moreover, Zhukovsky function maps conformally both the exterior and interior of the circle γ onto the complement of this curve. Consider another circle passing through $z = 1$ and touching γ at this point. Then Zhukovsky function maps the interior of these two circles on the domain (see Fig. 1), which represents the airfoil (profile of the airplane wing).

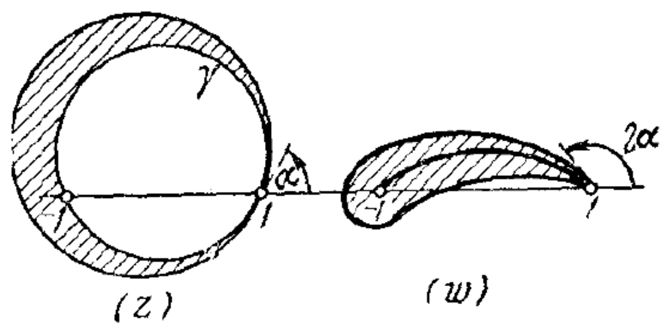

Fig. 1